Name: NEPTUN code:		**Point(s):** *Leave this field empty!*	

Useful constants: $h = 6.63\,10^{-34} Js, e = 1.6\,10^{-19}C, m_e = 9.1\,10^{-31}kg$, $L_A = 6.022\,10^{23} 1/mol$

1. In a 3D spherical quantum harmonic oscillator the energy of the 3rd excited state is $0.73eV$. What is the frequency ω? (**2 points**)

   Solution:

$$E_{n,m,l} = \hbar\omega \left(n + m + l + \frac{3}{2}\right), where \quad n, m, l = 0, 1, 2, ...$$

The energies of the states:
ground state : $E_0 = 1.5 \cdot \hbar\omega$,
First excited state: $E_1 = \hbar\omega(1 + 0 + 0 + 1.5)$,
2nd exc. state $E_2 = \hbar\omega(1 + 1 + 0 + 0.5)$
3rd excited state $E_3 = \hbar\omega(1 + 1 + 1 + 1.5)$

$$\omega = \frac{E_3}{\hbar\left(3 + \frac{3}{2}\right)} = \frac{0.73 \cdot 1.66\,10^{-19}}{6.63\,10^{-34} \cdot 4.5} = 2.46\,10^{14} Hz$$

2. Calculate the energy of the ground state of H, if the wavelength of the light emitted in transition $E_4 \rightarrow E_2$ is 486 nm (**2 points**)

   Solution:
   The wavelength of the emitted light is determined by the energy difference $E_4 - E_2$*:*

$$h\nu = h\,\frac{c}{\lambda} = \Delta E$$

Let's denote the ground state with E_1*. For H*

$$E_n = E_1 \frac{1}{n^2}$$

so

$$\Delta E = E_1 \left(\frac{1}{16} - \frac{1}{4}\right)$$

$$h\,\frac{c}{\lambda} = E_0 \left(\frac{1}{16} - \frac{1}{4}\right)$$

$$E_0 = \frac{h\,\frac{c}{\lambda}}{\left(\frac{1}{16} - \frac{1}{4}\right)} = 2.18\,10^{-18} J = -13.6eV$$

3. An electron is confined into a three dimensional cubic potential box with sides of $L = 3.6\mu m$. What is the wavelength of photons emitted during an electronic transition between level 3 and the ground state? How would this value change if the size of the box was doubled? (**2 points**)

Solution:
1D potential box: $L = n \cdot \lambda/2$, *where* $n = 1, 2, ...$ *So* $p_n = h/\lambda_n = h \cdot n/L$, *i.e.*

$$E_n = \frac{p^2}{2m_e} = \frac{h^2}{2m_e(2L)^2} n \qquad n = 1, 2, ...$$

For a 3D cubic potential box:

$$E_{n,m,l} = \frac{h^2}{8m_e L^2}(n^2 + m^2 + l^2) \qquad n, m, l = 1, 2, ...$$

The energy of the ground level is

$$E_{111} = 3\frac{h^2}{8m_e L^2}$$

For the 3rd level in 3 D

$$E_{122} = E_{212} = E_{221} = 9\frac{h^2}{2m_e L^2} = 3E_{111}$$

The energy of the emitted photon then is

$$E_{photon} = h\nu = \frac{hc}{\lambda} = \Delta E = 6\frac{h^2}{8m_e L^2} = 2.79\,10^{-26} J$$

$$\lambda = \frac{hc}{\Delta E} = 7.12m$$

If the size of the box is doubled the energy levels will be 4 times smaller and the phonon wavelength 4 times larger.

4. Sketch the conventional, primitive and Wigner-Seitz unit cells of a 2 dimensional fcc lattice (**2 points**)
   Solution:
5. The dispersion relation of electrons in a semiconductor valence band near the band edge is approximated by the following function:

$$E_v(k) = -6.048\,10^{-20}(k - 2.4510^8)^2 + 13 \qquad [eV]$$

In the same semiconductor the energy near the conduction band edge is

$$E_c(k) = 9.29\,10^{-20} k^2 + 13.7 \qquad [eV]$$

Determine the effective masses of the electrons and the holes. (**2 points**)
Solution:
The energy formula in J for electrons in the valence band:

$$E_{e,v}(k) = (-6.048\,10^{-20}(k - 2.4510^8)^2 + 13) \cdot 1.6\,10^{-19} \qquad [J]$$

In the conduction band:

$$E_{e,c}(k) = (9.29\,10^{-20} k^2 + 13.7) \cdot 1.6\,10^{-19} \qquad [J]$$

Therefore the hole dispersion relation in the valence band is:

$$E_{h,v}(k) = -E_{e,v}(k) = (6.048\,10^{-20}(k - 2.4510^8)^2 + 13) \cdot 1.6\,10^{-19} \qquad [J]$$

The definition of the effective mass:

$$\frac{1}{m_{eff}} = \frac{1}{\hbar^2}\frac{d^2E(k)}{dk^2}$$

From this:

$$\frac{1}{m_c} = \frac{2.98\,10^{-38}}{\hbar^2} = 1.74\,10^{30}\ kg^{-1} \qquad \Rightarrow \quad m_c = 3.74\,10^{-31}kg = 0.41\,m_e$$

$$\frac{1}{m_v} = \frac{1.94\,10^{-38}}{\hbar^2} = 2,67\,10^{30}\ kg^{-1} \qquad \Rightarrow \quad m_v = 5.74\,10^{-31}kg = 0.63\,m_e$$