2. Bose-Einstein statistic

Identical and indistinguishable particles, for the system of which the exclusion principle ↓ ⇝ there can be any number of particles in the same state
The wavefunction of the system is symmetric
Particles with integer spin (0, 1, ..) e.g. ^{4}He nucleus, H_2 ~
system of even number of half spin particles, photon

Let the degree of degeneracy of the E_i state be g_i
n_i particles can occupy these degenerate states in $(n_i + g_i - 1)!$ ways when the particles are distinguishable. For indistinguishable particles permutations of n_i and $g_i - 1$ give the same distribution
⇝ the number of distinguishable distributions is

$$\frac{(n_i + g_i - 1)!}{n_i!\,(g_i - 1)!}$$

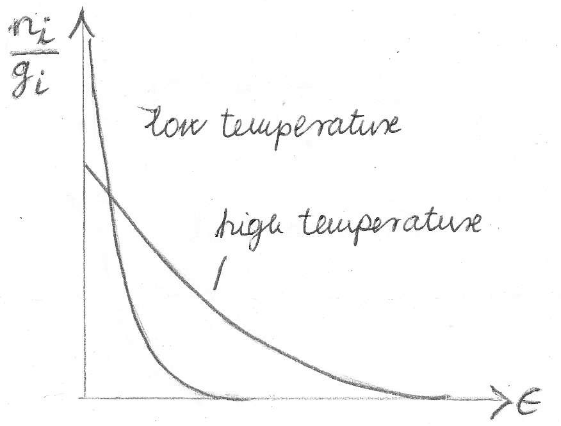
n_i particles
g_i states → $g_i - 1$ walls

The probability of $\{n_i\}$ distr.

$$P = \prod_i \frac{(n_i + g_i - 1)!}{n_i!\,(g_i - 1)!}$$

Its maximum corresponds to the statistical equilibrium

$$n_i = \frac{g_i}{e^{\alpha + \beta E_i} - 1}$$



$$\beta = \frac{1}{kT}$$

$$N = \sum_i n_i \rightsquigarrow \alpha$$

$\alpha \geq 0$, because $n_i \geq 0$

Population of lower energy levels is higher than for the Maxw. Boltzm. distribution

The photon "gas"

Electromagnetic radiation in a cavity in thermal equilibrium with the walls of the cavity (dinamic equilibrium: absorption and emission rates of the wall are identical)

Search: energy distribution of the electromagnetic radiation (spectrum)

Electromagnetic radiation in absorption and emission processes behaves like particles

photons $\left\{ \begin{array}{l} \text{emission processes behaves like particles} \\ \text{Energy: } h\nu \text{, momentum } h/\lambda \text{, indistinguishable,} \\ \text{with each other non-interacting particles} \\ \text{Any number of particles with given energy} \rightarrow \\ \underline{\text{bosons}} \text{ / angular momentum} = 1 \text{ (circ. pol.)} \end{array} \right.$

The particle number in the cavity $\neq$ const. since the walls can absorb or emit ~

α arbitrary in thermal equilibr. Let $\alpha = 0$

In statistical equilibrium

$$n_i = \frac{g_i}{e^{E_i/kT} - 1}$$

Possible energy states continuous

$$g_i \rightarrow g(E)\,dE$$

$$dn = \frac{g(E)\,dE}{e^{E/kT} - 1}$$ number of photons between E and $E + dE$

Density of states

$$g(E)\,dE = g(\nu)\,d\nu \qquad \text{since } E = h\nu$$

In a cavity with volume V the number of states with E energy → density of states

$$g(E) = \frac{4\pi V (2m^3)^{1/2}}{h^3} E^{1/2}$$

$$p = \sqrt{2mE} = \frac{h}{\lambda} \qquad \nu = \frac{c}{\lambda}$$

$$g(\nu) = \frac{4\pi V}{c^3} \nu^2$$

Electromagnetic wave - transverse → 2 polarizations

$$g(E)dE = g(\nu)d\nu = \frac{8\pi V}{c^3} \nu^2 d\nu \qquad \text{with this}$$

$$dn = \frac{8\pi V}{c^3} \frac{\nu^2 d\nu}{e^{h\nu/kT} - 1}$$

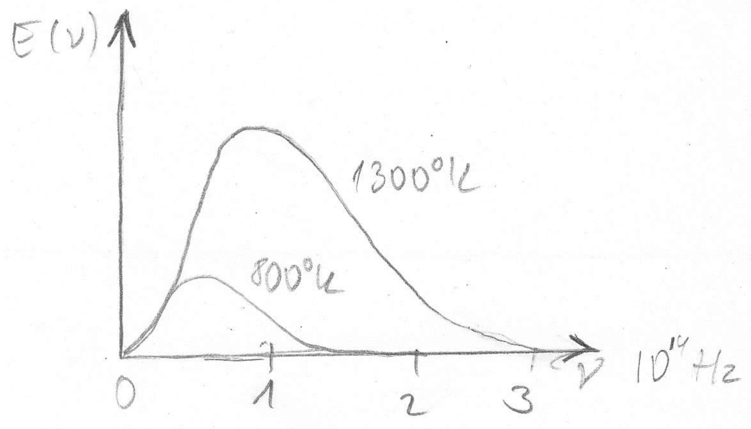
Energy of a photon $h\nu$, volume of cavity is V

The <u>energy density</u> $\quad E(\nu) = \frac{h\nu}{V} \frac{dn}{d\nu}$

$$\mathcal{E}(\nu) = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

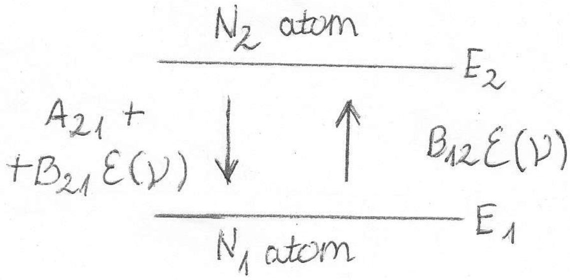
Planck's law for the blackbody radiation

Considering the electromagnetic radiation as system of particles following Bose-Einstein statistics we get a spectrum corresponding to experimental results!



Interaction of light and matter

How does material (walls of the cavity or a gas in the cavity) interacting with e.m. radiation in thermal equilibrium?



Model: 2 states

$\Delta E = E_2 - E_1$

$h\nu = \Delta E$ transition between 2 states

$\mathcal{E}(\nu)$ = energy density

The absorption probability per unit time

$$B_{12}\,\mathcal{E}(\nu)$$

$B_{12} = W(1 \to 2)$ transition probability per unit time and unit energy density

Interaction with particles at level E_2

$B_{21}\,\mathcal{E}(\nu)$ induced emission probability

A_{21} spontaneous — " — — " —

In unit time

No. of $N_2 \to N_1$ transitions $[A_{21} + B_{21}\,\mathcal{E}(\nu)]N_2$

No. of $N_1 \to N_2$ transitions $B_{12}\,\mathcal{E}(\nu)\,N_1$

$$\frac{dN_2}{dt} = \underbrace{B_{12}\,\mathcal{E}(\nu)N_1}_{\text{absorption}} - \underbrace{[A_{21} + B_{21}\,\mathcal{E}(\nu)]N_2}_{\text{emission}}$$

In thermal equilibrium $dN_2/dt = 0$ and the atoms $N_1/N_2 = e^{(E_2-E_1)/kT} = e^{h\nu/kT}$ Maxwell-Boltzm distribution

$B_{12}\,\mathcal{E}(\nu)e^{h\nu/kT} = A_{21} + B_{21}\,\mathcal{E}(\nu) \leadsto$

$$\mathcal{E}(\nu) = \frac{A_{21} + B_{12}}{e^{h\nu/kT} - B_{21}/B_{12}}$$

$\leftarrow$ Shape of spectrum $\equiv$ $\equiv$ Planck's law!

From Planck's law

$B_{12} = B_{21}$ $\quad A_{21} = \frac{8\pi\nu^3}{c^3} B_{21}$ ← probability of spontaneous emission

absorption prob. = ind. em. probability $(W(i \to k) = W(k \to i))$

$$\frac{\text{Spont. em. probab.}}{\text{Ind. em. probab.}} = \frac{A_{21}}{B_{21}\varepsilon(\nu)} = e^{\frac{h\nu}{kT}} - 1$$

$\frac{h\nu}{kT} >> 1$ induced emission insignificant (electronic transitions fr. above light)

$\frac{h\nu}{kT} << 1$ induced emission is significant (e.g. microwaves)

Induced emission: photon generated with identical frequency and phase with the incident photon → coherent

Spontaneous emission: incoherent

Principle of laser operation

$$\frac{\text{Prob. emission/unit time}}{\text{Prob. absorption/unit time}} = \frac{[A_{21} + B_{21}(\varepsilon(\nu))] N_2}{B_{12}\varepsilon(\nu) N_1} =$$

$$= \left(1 + \frac{A_{21}}{B_{21}(\varepsilon(\nu))}\right) \frac{N_2}{N_1}$$

If the system of atoms is not in thermal equilibrium

$N_2 > N_1$ is possible ⇝ emission > absorption

The medium amplifies the e.m. radiation

$N_2 > N_1$ inverse population ← with external pumping

Amplification — coherent

Amplifier + feedback ≡ oscillator

Feedback — optical resonator

Lasers

2 —— E_2 N_2

F = photon flux

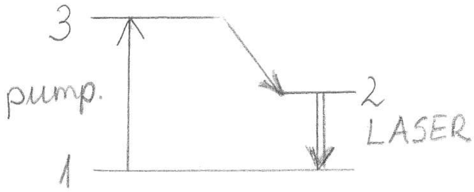
1 —— E_1 N_1

$$dF = \sigma F (N_2 - N_1)\, dz$$

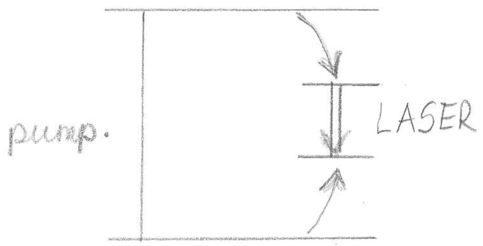
$$\frac{N_2}{N_1} = e^{-\frac{E_2 - E_1}{kt}} < 1 \rightsquigarrow dF < 0$$

$$\frac{N_2}{N_1} > 1 \rightsquigarrow dF > 0 \,!$$

Amplifier

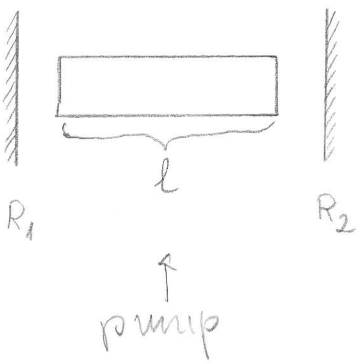


(impulse mode)



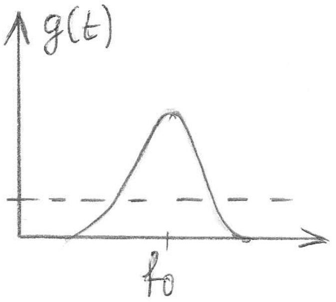
(continuous mode)

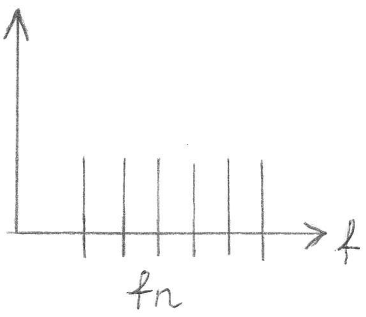
Oscillator



$$e^{\sigma (N_2 - N_1)\cdot l} \cdot R_1 \cdot R_2 > 1$$

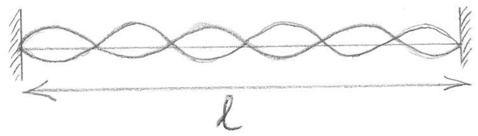
$$(N_2 - N_1) \geq \frac{\ln R_1 R_2}{\sigma l}$$

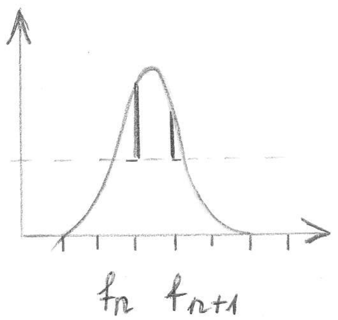




resonator modes

$f_n = n \cdot \frac{c}{2l}$





Gauss beams $I(r) \sim e^{-\frac{r^2}{w^2}}$

gaslasers : He-Ne, CO_2 ...

solid : Nd:YAG, Nd:glass

semiconductor : AlGaAs, GaAsP...

Coherence