Solid State Physics

$$\boxed{j_e = -j_h} \qquad j_{full} = 0$$

$$j_e = -\frac{e}{8\pi^3}\int \underline{v}(\underline{k})\, d^3k \quad \leftarrow \quad \int \text{occupied levels}$$

$$j_h = +\frac{e}{8\pi^3}\int \underline{v}(\underline{k})\, d^3k \quad \leftarrow \quad \int \text{unoccupied levels}$$

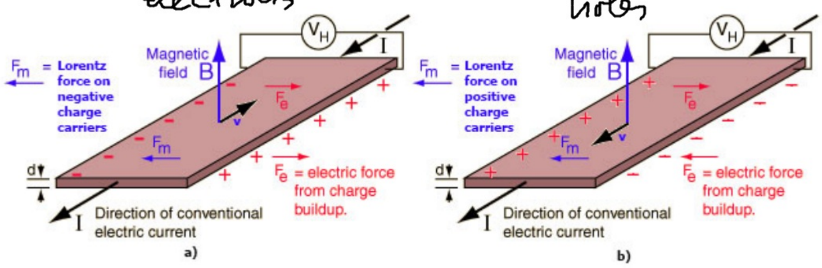
conductors metal

Experimental determination of sign of majority charge carriers: Hall effect

magnetic Lorentz force $\underline{F}_m = Q\, \underline{v} \times \underline{B}$

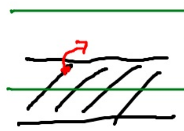
electrons holes

Hall probe →



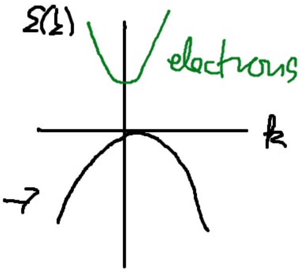
$Q = -e$ $\qquad$ $Q = +e$

Mg Cd



conduction band

valence band



$$m_{eff} = \frac{1}{\hbar^2}\,\frac{1}{\left(\frac{d^2\varepsilon}{dk^2}\right)}$$

$v = \mu E$ $\qquad$ $\mu = \dfrac{e\tau}{m_{eff}}$

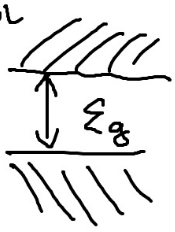
mobility

$\varepsilon \approx ak^2 + bk + c$

a larger → m_{eff} smaller

⇒ μ larger

Insulator



$\varepsilon_g \gg k_B T$ $\qquad$ $\sim 6-10\,eV$

Diamond $\varepsilon_g = 5.5\,eV$

$T \sim 300\,K$

$k_B T = \boxed{0.025}8\ eV$

$$e^{-\varepsilon_g/k_BT}$$

$\boxed{\text{Semiinductors}}$

$\mathcal{E}_g \approx 1\,eV$ $\quad$ $\mathcal{E}_g^{(Si)} = 1.1\,eV$

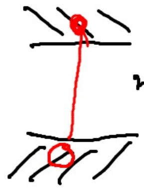
$\mathcal{E}_g^{(Ge)} = 0.7\,eV$

Si $\quad \rho(300K) = 3.32 \cdot 10^{-19} \qquad \rho(450K) = 1.44 \cdot 10^{-6}$

$\rho_{Si} \approx 10^{22} \frac{\text{ata}}{cm^3}$ $\qquad \rho(450K)/\rho(300K) \approx \underline{\underline{4 \cdot 10^{12}}}$!

$R_{(S}$ $\qquad R_{300} \gg R_{450}$ $\qquad$ negative thermal coefficients

$R \sim \Delta t$



negative $\to$ $\tau_n(s)$ mean lifetime of electrons in the conduction band

$\tau_p\ (s')$ mean lifetime of holes in the valence band

positive $\to$

$$\tau_n(s) = \tau_p(s')$$

electrons $\to n_c \leftarrow$ density of charge carriers in the conduction band (electrons)

$p_v \leftarrow$ density of charge carriers in the valence band (holes)

$n_i := n_c = p_v$

$\uparrow$ intrinsic density of charge carriers

intrinsic semiconductor (not doped)

$$j = \underset{\text{cond. b.}}{j_e} + \underset{\text{valence b.}}{j_h}$$

minority / majority charge carriers

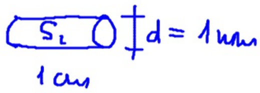
$\langle \underline{v} \rangle = \mu \underline{E}$ $\qquad j = -n_c e \langle \underline{v}_e \rangle + p_v e \langle \underline{v}_h \rangle$

$\mu = \frac{\pm e \tau}{m_{eff}}$ $\qquad j = n_i e (\mu_e + \mu_h) \underline{E}$

$$\boxed{\sigma = n_i e (\mu_e + \mu_h)}$$

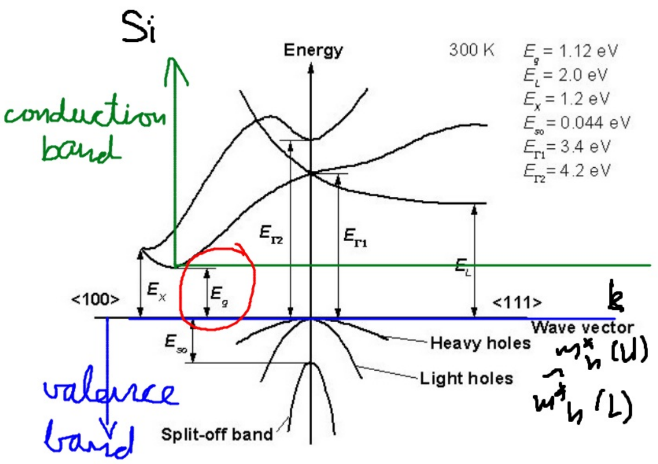
Example: Si $\quad n_i = 1.5 \cdot 10^{16} \frac{1}{m^3} \quad \mu_e = 0.13 \frac{m^2}{Vs} \quad \mu_h = 0.05 \frac{m^2}{Vs}$

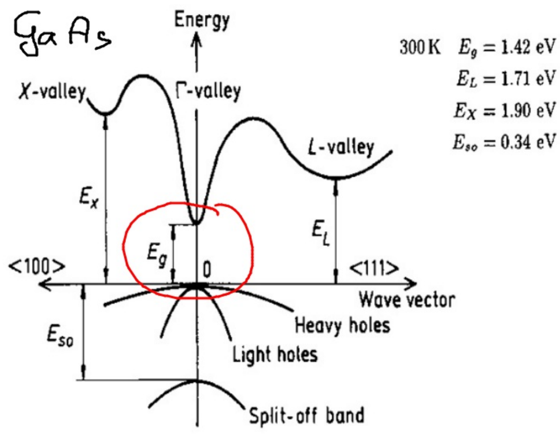
$\sigma = ?$ $\quad \sigma = 1.5 \cdot 10^{16} \cdot 1.6 \cdot 10^{-19} (0.13 + 0.05) = 4.33 \cdot 10^{-4} \frac{1}{\Omega m}$ $\quad \left(\frac{1}{m^{\cancel{3}}} \cdot A\cancel{s} \cdot \frac{m^{\cancel{2}}}{V\cancel{s}} = \frac{1}{\frac{V}{A} \cdot m} \right.$



$R = ?$ $\qquad R = \rho \frac{l}{A} = \frac{l}{\sigma A} = \frac{10^{-2}}{4.33 \cdot 10^{-4} (0.5 \cdot 10^{-3})^2 \cdot \pi} = 29.4\ M\Omega$

Real band structures:

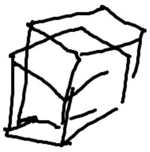




Si indirect gap semiconductor

GaAs direct gap

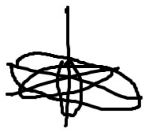
fcc



$\frac{1}{4}d$

$$\varepsilon_{cond}(\underline{k}) = E_c + \frac{\hbar^2}{2}\left(\frac{k_1^2}{m^*_{e,1}} + \frac{k_2^2}{m^*_{e,1}} + \frac{k_3^2}{m^*_{e,2}}\right)$$

$*$ → effective mass   e – electron



$$\varepsilon_{valenc}(\underline{k}) = E_v - \frac{\hbar^2}{2}\left(\frac{k_1^2}{m^*_{h,1}} + \frac{k_2^2}{m^*_{h,1}} + \frac{k_3^2}{m^*_{h,2}}\right)$$

pot. box V_0 = const

$$V(x) = \begin{cases} 0 & \text{inside} \\ \infty & \text{outside} \end{cases}$$

ε

intrinsic semiconductor

conduction	electrons	$V_0 = \varepsilon_c$	$\varepsilon \Rightarrow \varepsilon - \varepsilon_c > 0$
	holes	$V_0 = \varepsilon_v$	$\varepsilon \Rightarrow \varepsilon_v - \varepsilon > 0$

electrons

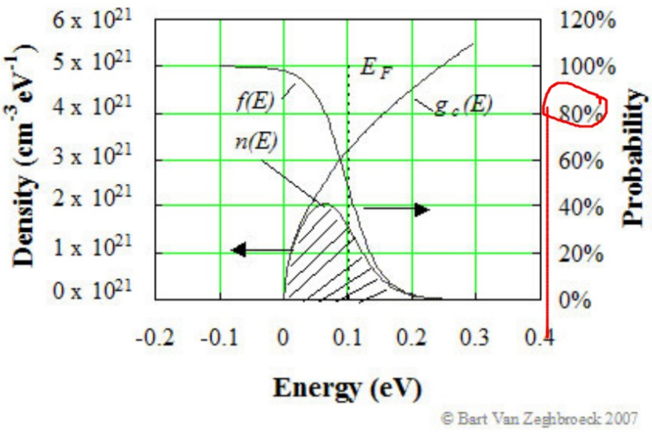
	rel. eff.mass	Si	Ge
longitudinal	m_l/m_e	0.98	1.59
transverse	m_t/m_e	0.19	0.0815
for dens.of.states	m_c/m_e	0.36	0.22
for conduct.	m_{cc}/m_e	0.26	0.12 ← m_e

holes

	rel. eff.mass	Si	Ge
heavy	m_h/m_e	0.49	0.33
light	m_{lp}/m_e	0.16	0.043
split-off band	m_{so}/m_e	0.24	0.084
dens.of.states	m_v/m_e	0.81	0.34 ← m_h

$$g_c(\varepsilon) = \frac{8\pi\sqrt{2m_e^3}}{\hbar^3}\sqrt{\varepsilon - \varepsilon_c} \qquad \varepsilon \geq \varepsilon_c$$

$$g_v(\varepsilon) = \frac{8\pi\sqrt{2m_h^3}}{\hbar^3}\sqrt{\varepsilon_v - \varepsilon} \qquad \varepsilon \leq \varepsilon_v$$



$$n(\varepsilon) = f(\varepsilon)\, g_c(\varepsilon)$$

$$\langle O \rangle = \int_0^{\infty} O(\varepsilon)\, f_{F-D}(\varepsilon)\, g(\varepsilon)\, d\varepsilon$$

for n $\quad O \equiv 1$

cond. band $\quad f_{F-D}^{(\text{electron})} = \dfrac{1}{e^{(\varepsilon-\varepsilon_F)/k_BT}+1}$

valence band $\quad f_{F-D}^{(\text{holes})} = 1 - f_{F-D}^{(\text{electron})}$

$$f_{F-D}^{(\text{holes})} = 1 - \frac{1}{e^{(\varepsilon-\varepsilon_F)/k_BT}+1} = \frac{e^{(\varepsilon-\varepsilon_F)/k_BT} + \cancel{1} - \cancel{1}}{e^{(\varepsilon-\varepsilon_F)/k_BT}+1} = \frac{1}{e^{-(\varepsilon-\varepsilon_F)/k_BT}+1}$$

$$n_c(T) = \int_{\varepsilon_c}^{\infty} g_c(\varepsilon) \frac{d\varepsilon}{e^{(\varepsilon-\varepsilon_F)/k_BT}+1}$$

$$p_v(T) = \int_{-\infty}^{\varepsilon_v} g_v(\varepsilon) \frac{1}{e^{-(\varepsilon-\varepsilon_F)/k_BT}+1}\, d\varepsilon$$

$\boxed{\text{IF}}$ non-degenerate semiconductor ($\varepsilon_g \gg 3 k_B T$)

$$n_c(T) = N_c\, e^{-(\varepsilon_c-\varepsilon_F)/k_BT}$$

$$p_v(T) = P_v\, e^{-(\varepsilon_F-\varepsilon_v)/k_BT}$$

where
$$N_c(T) = 2.5 \cdot 10^{19} \cdot \left(\frac{m_c^*}{m_e}\right)^{3/2} \left(\frac{T}{300\text{K}}\right)^{3/2}$$

$$P_v(T) = 2.5 \cdot 10^{19} \left(\frac{m_h^*}{m_e}\right)^{3/2} \left(\frac{T}{300\text{K}}\right)^{3/2} \qquad (*)$$

example

	Si	Ge
m_e^*/m_e	0.36	0.22
m_h^*/m_h	0.81	0.34
$N_c(300K)$	$2.81 \cdot 10^{19}$	$1.02 \cdot 10^{19}$
$P_v(300K)$	$1.83 \cdot 10^{19}$	$5.64 \cdot 10^{18}$

$$n_i^2 = \overbrace{n_c \cdot p_v}^{\text{true for all semiconductors}} = N_c P_v \, e^{-(\varepsilon_c - \varepsilon_v)/k_B T} = N_c P_v \, e^{-\varepsilon_g / k_B T}$$

$n_c = n_i$, $p_v = n_i$ — only true for intrinsic semic.

$$\boxed{\text{mass-action law for semiconductors}}$$

$$N_c(T) e^{-\frac{(\varepsilon_c - \varepsilon_F)}{k_B T}} = P_v(T) e^{-\frac{\varepsilon_F - \varepsilon_v}{k_B T}} \Rightarrow \frac{N_c(T)}{P_v(T)} = e^{-\frac{\varepsilon_F - \varepsilon_v + \varepsilon_F - \varepsilon_c}{k_B T}}$$

$$-2\varepsilon_F + (\varepsilon_c + \varepsilon_v) = k_B T \cdot \frac{N_c(T)}{P_v(T)}$$

$$\varepsilon_F = \frac{1}{2}(\varepsilon_c + \varepsilon_v) + \frac{1}{2} k_B T \ln\left(\frac{P_v}{N_c}\right)$$

from (*)

$$\boxed{\varepsilon_F = \varepsilon_v + \frac{1}{2}\varepsilon_g + \frac{3}{4} k_B T \ln\left(\frac{m_h^*}{m_e^*}\right)}$$

if $m_h^* = m_e^* \Rightarrow \varepsilon_F = \underset{\substack{\uparrow \\ \text{usually taken to } 0}}{\varepsilon_v} + \frac{1}{2}\varepsilon_g$ in the middle of ε_g

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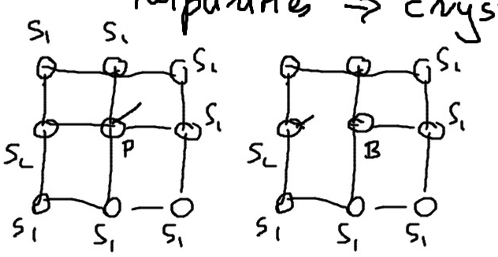
Extrinsic semiconductors

impurities → crystal



group IV semicond.

Si, Ge 4

Group III B

Group V P

Ionization energy:

H in vacuum

$$\varepsilon_{ion}^{(H)} = \frac{e^4 m_e^2}{32\pi^2 \epsilon_0^2 \hbar^2} \cdot \frac{1}{1^2} = \frac{e^4}{32\pi^2 \hbar^2} \frac{m_e^2}{\epsilon_0^2} = 13.6\,eV$$

$$r_{Bohr} = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} = \frac{4\pi\hbar^2}{e^2} \frac{\epsilon_0}{m_e} = 0.0529\,nm$$

in a material with m_e^* and relative permittivity ϵ_r is used

$$\varepsilon_{ion} = 13.6\,eV \cdot \frac{m_e^*}{m_e} \cdot \frac{1}{\epsilon_r^2}$$

$$r_0 = r_{Bohr} \cdot \frac{m_e}{m^*} \epsilon_r$$

Example Si: $\epsilon_r = 11.7$ $\to \varepsilon \approx 12.6 \cdot 0.26 / 11.7^2 = 0.0258\,eV$

$m^* = m_{cc} = 0.26\, m_e$ — / silicon

$r_D = r_{Bohr} \cdot \frac{11.7}{0.26} = 2.38\ nm$

How many electrons from donor atoms are excited?

$$\left(\underline{D} = \epsilon_r \epsilon_0 \underline{E} \right)$$

Cond. $\mathcal{E}_c - \mathcal{E}_D = 0.0258\ eV$

$\mathcal{E}_c$

$-\mathcal{E}_D$

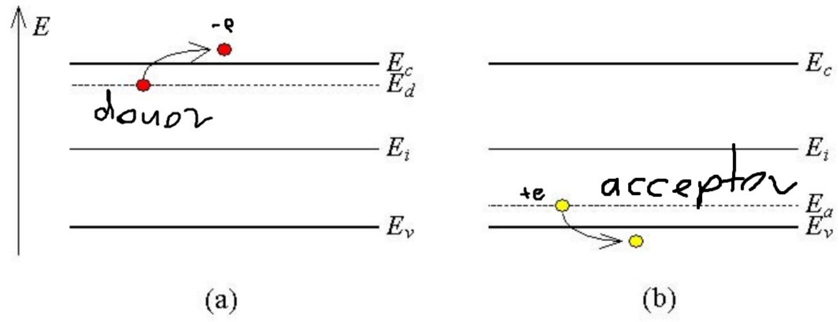
$\mathcal{E}_g = 1.1\ eV$

$\mathcal{E}_v$

Valence

$k_B T (300K) = 0.0258\ eV \Rightarrow 36.8\%$ ionized

in intrinsic Si only $3 \cdot 10^{-17}\%$!



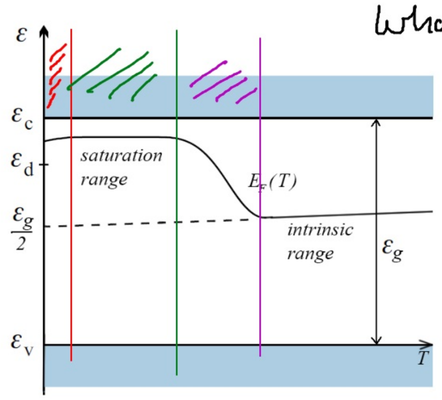
(a) (b)

$\mathcal{E}_F(T)$?

from N_d

all ionized

from valence band too



What is $\mathcal{E}_F$ here? metals: at 0K all levels below it is occupied above it empty

semiconductors / insulators: defined by $\mathcal{E}_c, \mathcal{E}_v$ and f_{F-D} and $g(\mathcal{E})$

$\boxed{\text{doped semiconductors}}$ — extrinsic semic.

crystal neutral: $\Sigma Q = 0$ $\quad \Sigma Q = \underbrace{N^+ + p_v}_{+} \underbrace{- N^- - n_c}_{-} = 0$

n-type the electrons
p-type the holes
are the majority charge carriers

$n_c, p_v \qquad n_c \neq n_i \qquad p_v \neq n_i$

mass-action law for semiconductors

n-type $\quad n_c \approx N_d - N_a$

p-type $\quad p_v \approx N_a - N_d$

$$n_c \cdot p_v = n_i^2$$