

Problems in Solid State Physics for Electrical and Chemical Engineers

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The problems presented here together with their solutions help the deeper understanding of the lectures. Similar problems may be given in the tests. We tried to give a very detailed solution for every one of them, however in the tests you need not give so much details. This material will be refreshed from time to time, so please check the date on the title page!

Chapter 1

Formulas used in the solutions

1.1 Solid State Physics

Reciprocal Lattice

There exists an algebraic formula between the $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ primitive reciprocal lattice vectors and the three primitive direct lattice vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$

$$\begin{aligned}\mathbf{b}_1 &= 2\pi \frac{\mathbf{a}_2 \times \mathbf{a}_3}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)} \\ \mathbf{b}_2 &= 2\pi \frac{\mathbf{a}_3 \times \mathbf{a}_1}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)} \\ \mathbf{b}_3 &= 2\pi \frac{\mathbf{a}_1 \times \mathbf{a}_2}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}\end{aligned}\tag{1.1.1}$$

Effective Mass

The effective masses of electrons and holes in Si and Ge are in Table 1.1. The effective mass for conductivity of electrons m_{cc} is what we must use for mobility or diffusion constant calculations. For cubic isotropic semiconductors with anisotropic dispersion relations minima (again e.g. Si and Ge), one has to sum over the effective masses in the different minima along the equivalent directions. The resulting effective mass for bands which have ellipsoidal constant energy surfaces is given by:

$$m_{cc} = \frac{3}{\frac{1}{m_l} + \frac{1}{m_l} + \frac{1}{m_t}}$$

electrons			
	rel. eff.mass	Si	Ge
longitudinal	m_l/m_e	0.98	1.59
transverse	m_t/m_e	0.19	0.0815
dens.of.states	m_c/m_e	0.36	0.22
conduct.	m_{cc}/m_e	0.26	0.12
holes			
	rel. eff.mass	Si	Ge
heavy	m_h/m_e	0.49	0.33
light	m_{lp}/m_e	0.16	0.043
split-off band	m_{so}/m_e	0.24	0.084
dens.of.states	m_v/m_e	0.81	0.34

Table 1.1: Effective masses in Si and Ge

Density of State Factors in Semiconductors

$$N_c(T) = 2.5 \cdot 10^{19} \left(\frac{m_c}{m_e} \right)^{\frac{3}{2}} \left(\frac{T}{300K} \right)^{\frac{3}{2}} \quad (1.1.2)$$

$$P_v(T) = 2.5 \cdot 10^{19} \left(\frac{m_h}{m_e} \right)^{\frac{3}{2}} \left(\frac{T}{300K} \right)^{\frac{3}{2}} \quad (1.1.3)$$

Primitive Vectors for the bcc lattice

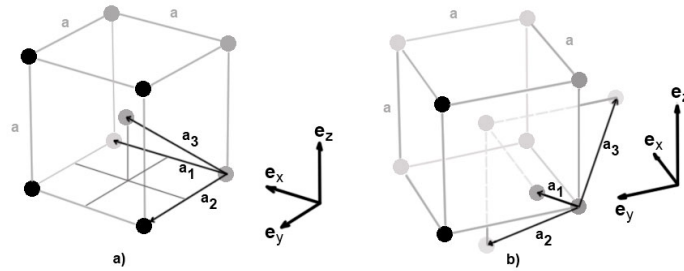


Figure 1.1: Two different primitive vector sets for the bcc lattice.

$$\begin{aligned} \text{a) } & \mathbf{a}_1 = a\mathbf{e}_x, \quad \mathbf{a}_2 = a\mathbf{e}_y, \quad \mathbf{a}_3 = a\mathbf{e}_z \\ \text{b) } & \mathbf{a}_1 = \frac{a}{2}(\mathbf{e}_y + \mathbf{e}_z - \mathbf{e}_x), \quad \mathbf{a}_2 = \frac{a}{2}(\mathbf{e}_z + \mathbf{e}_x - \mathbf{e}_y), \quad \mathbf{a}_3 = \frac{a}{2}(\mathbf{e}_x + \mathbf{e}_y - \mathbf{e}_z) \end{aligned}$$

drift velocity:

$$\mathbf{v}_{drift} = \mu\mathbf{E} \quad (1.1.4)$$

where the constant of proportionality is the mobility μ of the electrons

$$\mu = \left(\frac{e\tau}{m_e} \right) \quad (1.1.5)$$

Conductivity vs. Relaxation Time

$$\mathbf{j} (= \sigma \mathbf{E}) = \left(\frac{ne^2\tau}{m_e} \right) \mathbf{E}$$

and

$$\sigma = \frac{ne^2\tau}{m_e} \quad (1.1.6)$$

One way to determine the value of τ is from (1.1.6), where the density n of conduction electrons is

$$n = L_A \frac{Z\rho_m}{A} \quad (1.1.7)$$

Current Density in Semiconductors

$$\mathbf{j} = e(n_c \mu_e + p_v \mu_h) \mathbf{E} \quad (1.1.8)$$

$$\begin{aligned} \mathbf{j} &= n_i e \langle \mathbf{v} \rangle = n_i e (\mu_e + \mu_h) \mathbf{E} \quad \text{and} \\ \mathbf{j} &= \sigma \mathbf{E} \end{aligned}$$

$$\sigma = n_i e (\mu_e + \mu_h) \quad (1.1.9)$$

Fermi Energies

Metal	Li	Na	K	Rb	Cs	Cu	Ag	Au	Mg	Al
\mathcal{E}_F (eV)	4.7	3.1	2.1	1.8	1.5	4.1	5.5	5.5	7.3	11.9

Table 1.2: Fermi energies of some metals

Any of these energy states can be occupied by 2 electrons with opposite spins. If each atom contribute s conduction electrons to the crystal then at $T = 0 K$ all possible energy states below

$$\mathcal{E}_F (= \mathcal{E}_{max,occupied}) = \frac{\hbar^2 \pi^2 s}{2m_e a^2} \quad (1.1.10)$$

will be occupied and all levels above it will be empty. \mathcal{E}_F is called the Fermi energy of the system. We denote the corresponding k value with k_F and call it the Fermi wave vector.

The Fermi-Dirac distribution function

$$f_{FD}(\mathcal{E}) = \frac{1}{e^{(\mathcal{E}-\mathcal{E}_F)/k_B T} + 1} \quad (1.1.11)$$

The number of electrons ($N_e = N \cdot s$):

$$N_e = 2 \frac{k_F^3}{6\pi^2} V \quad \Rightarrow \quad n(\equiv \frac{N_e}{V}) = \frac{k_F^3}{3\pi^2} \quad (1.1.12)$$

When $s = 2$ the length of the Fermi wave vector will be $2\pi/a$ and the allowed k values will satisfy

$$k \in \left[-\frac{\pi}{a}, \frac{\pi}{a} \right]$$

For n-type semiconductors ($N_d^+ > N_a^-$ where $N_d^+ \approx N_d$ and $N_a^- \approx N_a$):

$$n_c \approx (N_d - N_a) \quad p_v \approx \frac{n_i^2}{N_d - N_a} \quad (1.1.13)$$

while for p-type semiconductors ($N_a > N_d$):

$$p_v \approx (N_a - N_d) \quad n_c \approx \frac{n_i^2}{N_a - N_d} \quad (1.1.14)$$

Electron and Hole Densities

$$n_c(T) = N_c e^{-\frac{\mathcal{E}_c - \mathcal{E}_F}{k_B T}} \quad (1.1.15a)$$

$$p_v(T) = P_v e^{-\frac{\mathcal{E}_F - \mathcal{E}_v}{k_B T}} \quad (1.1.15b)$$

where N_c and P_v are the effective density of states in the conduction and valence bands respectively:

$$N_c(T) = 2 \cdot \left(\frac{2\pi m_e k_B T}{h^2} \right)^{\frac{3}{2}} \quad (1.1.16)$$

$$P_v(T) = 2 \cdot \left(\frac{2\pi m_h k_B T}{h^2} \right)^{\frac{3}{2}} \quad (1.1.17)$$

In doped semiconductors:

$$n_c^{(d)}(T) = n_i e^{-\frac{\mathcal{E}_F^{(d)} - \mathcal{E}_F}{k_B T}} \quad \text{for an n-type semiconductor} \quad (1.1.18a)$$

$$p_v^{(a)}(T) = n_i e^{-\frac{\mathcal{E}_F - \mathcal{E}_F^{(a)}}{k_B T}} \quad \text{for a p-type semiconductor} \quad (1.1.18b)$$

Extents of the Depletion Rangion

$$d_n = \sqrt{\frac{N_a}{N_d(N_d + N_a)} \frac{\epsilon \Delta \varphi}{2e}} \quad (1.1.19)$$

$$d_p = \sqrt{\frac{N_d}{N_a(N_d + N_a)} \frac{\epsilon \Delta \varphi}{2e}}$$

$$\mathcal{E}_F^{(d)} = \frac{1}{2} (\mathcal{E}_c + \mathcal{E}_{donor}) + \frac{3}{4} k_B T \ln \left(\frac{m_h}{m_c} \right) \quad (1.1.20a)$$

$$\mathcal{E}_F^{(a)} = \frac{1}{2} (\mathcal{E}_v + \mathcal{E}_{acceptor}) + \frac{3}{4} k_B T \ln \left(\frac{m_h}{m_c} \right) \quad (1.1.20b)$$

Donor and Acceptor levels

acceptor in	B	Al	Ga	In	Te
Si	0.046	0.057	0.065	0.16	0.26
Ge	0.01	0.01	0.061	0.011	0.01
donor in	P	As	Sb	Bi	
Si	0.046	0.057	0.065	0.16	.
Ge	0.01	0.01	0.061	0.011	.

Table 1.3: Shallow donor and acceptor levels in Si and Ge in eV.

Built-In Potential

$$e\Delta\varphi = k_B T \ln \left(\frac{N_d N_a}{n_i^2(T)} \right) \quad (1.1.21)$$

Mass-Action in Doped Semiconductors

For intrinsic semiconductors

$$n_c(T) = p_v(T) \quad (1.1.22)$$

$$n_i \equiv n_c(T) = p_v(T) = \sqrt{N_c(T)P_v(T)} e^{-\frac{\epsilon_g}{2k_B T}} \quad (1.1.23)$$

built-in potential

$$\Delta\varphi = \frac{1}{e} k_B T \ln \left(\frac{N_d N_a}{n_i^2} \right) \quad (1.1.24)$$

Curie-Weiss law

$$\mathcal{M} = \frac{C}{T - \lambda C} B \quad (1.1.25)$$

$$\chi = \frac{C}{T - \lambda C} = \frac{C}{T - T_c} \quad (1.1.26)$$

Material	T_c (K)
Co	1388
Fe	1043
MnBi	630
Ni	627
MnSb	587
CrO2	386
MnAs	318
Gd	292
Dy	88
EuO	69

Table 1.4: Curie temperatures of some crystalline materials
(Source: Wikipedia)

Problem 1.

An infinite number of points are aligned along a line so that the distance between the n -th and $(n + 1)$ -th points is $d_n = 7.8 [(n \bmod 2) \cdot 0.21 + (1 - n \bmod 2) \cdot 0.17]$ nm, where $a \bmod b$ is the remainder of the integer division a/b . May these points describe a one dimensional linear “crystal” and if they do then what do the neighboring points correspond to or if they do not then why not?

Solution:

If n is an even number then $n \bmod 2$ is 0, if it is an odd number then it is 1. From the definition above the distances between consecutive points are $d_1 = 7,8 \cdot 1,7 = 1,33 \text{ nm}$ when n is even and $d_2 = 7,8 \cdot 2,1 = 1,64 \text{ nm}$ when n is odd. These distances are repeated, which means this structure has a translational symmetry, i.e. it may correspond to a crystal. These points may refer to atoms in a diatomic crystal with a two atom basis and a base vector of $d_1 + d_2 = 2,97 \text{ nm}$

Problem 2.

What are the coordination numbers for an sc, a bcc and an fcc lattice?

Solution:

To calculate the coordination number you may select any lattice point. We select the ones easiest to calculate the coordinates for.

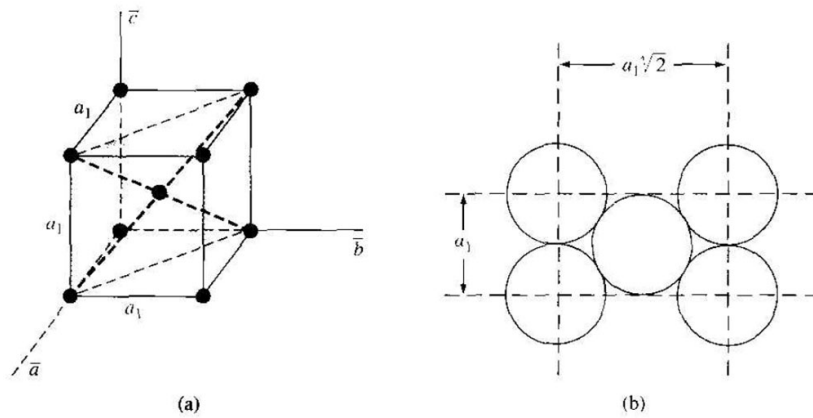
- sc - 6, because any corner point has 2 nearest neighbors in all 3 orthogonal planes that go through it
- fcc - 12, because any corner point has 4 nearest neighbors in all 3 orthogonal planes that go through it
- bcc - 8, because the center point has 8 neighbors

Problem 3.

Calculate the surface density of atoms in a bcc crystal with lattice constant $a = 0,5 \text{ nm}$ ($= 5 \text{ \AA}$), if the surface plane cuts the cells diagonally so that it is perpendicular to the plane of the a_1 and a_2 vectors¹.

Solution:

¹i.e. it is a (110) plane - see Miller indices below

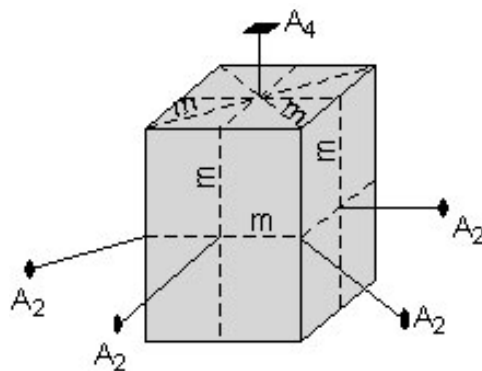


The plane in question goes through 4 corner atoms and the middle atom of the cell. Only 1/4th of each of the cross sections of the corner atoms belong to our cell, while the cross section of the middle atom is completely inside it. Therefore the number of atoms on this plane is 2. The area of the plane is $a \cdot a\sqrt{2}$, so the density of atoms is

$$\frac{2}{a^2\sqrt{2}} = \frac{\sqrt{2}}{(0.5 \cdot 10^{-9})^2} = 5.66 \cdot 10^{18} \frac{\text{atoms}}{\text{m}^2}$$

Problem 4.

Enumerate the symmetries the following “crystal” has.



Solution:

This “crystal” has the following symmetry elements:

- 1 - 4-fold rotation axis (A_4)
- 4 - 2-fold rotation axes (A_2), 2 cutting the faces and 2 cutting the edges
- 5 mirror planes (m), 2 cutting across the faces, 2 cutting through the edges, and one cutting horizontally through the center.

- There is a center of symmetry (i) in the center of the prism (not shown).

Problem 5.

Is the hexagonal lattice in Fig.?? a Bravais lattice? Why? If it is not then create the equivalent Bravais lattice from it!

Solution:

The hexagonal lattice in Fig.?? is not a Bravais lattice, because the orientation of the surrounding points is different when viewed from points P and Q.

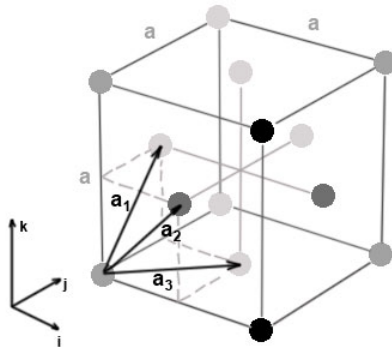
Select PQ and the equivalent other point pairs as a basis and select the points at the middle of these pairs as points of the point lattice. In the unnumbered Figure the blue lattice points form a Bravais lattice.

Problem 6.

Prove that the reciprocal lattice of an fcc lattice is a bcc lattice!

Solution:

Start with the following selection of primitive fcc lattice vectors:



Then the 3 primitive vectors in (1.1.1) are

$$\begin{aligned}
 \mathbf{a}_1 &= \frac{a}{2}(\mathbf{i} + \mathbf{k}) \\
 \mathbf{a}_2 &= \frac{a}{2}(\mathbf{i} + \mathbf{j}) \\
 \mathbf{a}_3 &= \frac{a}{2}(\mathbf{j} + \mathbf{k})
 \end{aligned}
 \tag{1.1.27}$$

Determine first the denominator in (1.1.1), which is the volume of the primitive cell:

$$\begin{aligned}
 \mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3) &= \frac{a}{2}(\mathbf{i} + \mathbf{k}) \cdot \left(\frac{a}{2}(\mathbf{i} + \mathbf{j}) \times \frac{a}{2}(\mathbf{j} + \mathbf{k}) \right) \\
 &= \frac{a^3}{8}(\mathbf{i} + \mathbf{k}) \cdot ((\mathbf{i} + \mathbf{j}) \times (\mathbf{j} + \mathbf{k})) \\
 &= \frac{a^3}{8}[\mathbf{i} \cdot (\mathbf{i} \times \mathbf{j}) + \mathbf{i} \cdot (\mathbf{i} \times \mathbf{k}) \\
 &\quad + \mathbf{i} \cdot (\mathbf{j} \times \mathbf{k}) + \mathbf{k} \cdot (\mathbf{i} \times \mathbf{j}) \\
 &\quad + \mathbf{k} \cdot (\mathbf{i} \times \mathbf{k}) + \mathbf{k} \cdot (\mathbf{j} \times \mathbf{k})] \\
 &= \frac{a^3}{8}(\mathbf{i} \cdot (\mathbf{j} \times \mathbf{k}) + \mathbf{k} \cdot (\mathbf{i} \times \mathbf{j}))
 \end{aligned}$$

here we used that \mathbf{i} , \mathbf{j} and \mathbf{k} are perpendicular to each other. Furthermore

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}, \mathbf{j} \times \mathbf{k} = \mathbf{i} \quad \text{and} \quad \mathbf{k} \times \mathbf{i} = \mathbf{j}$$

Therefore

$$\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3) = \frac{a^3}{4}$$

Now work with the numerators using the same formulas for \mathbf{i} , \mathbf{j} and \mathbf{k} :

$$\mathbf{a}_2 \times \mathbf{a}_3 = \frac{a^2}{4}(\mathbf{k} - \mathbf{j} + \mathbf{i}) \mathbf{a}_3 \times \mathbf{a}_1 = \frac{a^2}{4}(\mathbf{i} - \mathbf{k} + \mathbf{j}) \mathbf{a}_1 \times \mathbf{a}_2 = \frac{a^2}{4}(\mathbf{j} - \mathbf{i} + \mathbf{k})$$

Which gives us the reciprocal base vectors:

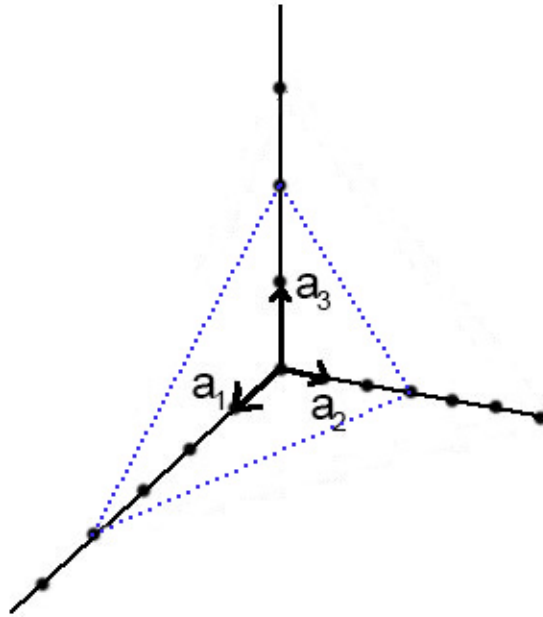
$$\begin{aligned}
 \mathbf{b}_1 &= \frac{2\pi}{a}(\mathbf{k} - \mathbf{j} + \mathbf{i}) \\
 \mathbf{b}_2 &= \frac{2\pi}{a}(\mathbf{i} - \mathbf{k} + \mathbf{j}) \\
 \mathbf{b}_3 &= \frac{2\pi}{a}(\mathbf{j} - \mathbf{i} + \mathbf{k})
 \end{aligned} \tag{1.1.28}$$

Compare (1.1.28) with vectors in Fig. 1.1 to see that we, in fact got the primitive vectors of a bcc lattice. The only difference is that the length is now, $\frac{2\pi}{a}$.

Because the reciprocal lattice of a reciprocal lattice is the original (direct) lattice, we also proved that the reciprocal lattice of a bcc lattice is an fcc lattice.

Problem 7.

Determine the Miller indexes for the plane on the figure!



Solution:

The intersections with the three axes are at $4a_1$, $3a_2$ and $2a_3$. Then the inverse intercepts in lattice vector units are:

$$\frac{1}{4}, \frac{1}{3}, \frac{1}{2}$$

To get integer numbers we have to calculate the lowest common denominator (smallest common multiple) of this fraction, which is 12. Multiplying each fraction with 12 gives the three Miller indexes: **(346)**

Problem 8.

In the previous example let the length of all the three base vectors of the direct lattice to be $a = 5 \text{ nm}$. Determine the distance of lattice planes (346).

Solution:

The three basis vectors are of the same length and they are perpendicular to each other. Therefore the same is true for the reciprocal base vectors. By substitution into (1.1.1) this length is $b = \frac{2\pi}{a} = 1.26 \text{ nm}^{-1}$. The length of the reciprocal vector perpendicular to the (346) planes is

$$|g_{346}| = \sqrt{3^2 + 4^2 + 6^2} \cdot \frac{2\pi}{a} = \sqrt{61} \cdot \frac{2\pi}{a} (= \sqrt{61} \cdot 1.26 = 9.81 \text{ nm}^{-1})$$

The distance of the planes (346) therefore is

$$\underline{\underline{d_{346}}} = \frac{2\pi}{|g_{346}|} = \frac{a}{\sqrt{61}} = \underline{\underline{0.64 \text{ nm}}}$$

Problem 9.

The 3 base vectors of a crystal are orthogonal and 2.3, 3.4 and 4.5 nm long. What is the distance of lattice planes (211)?

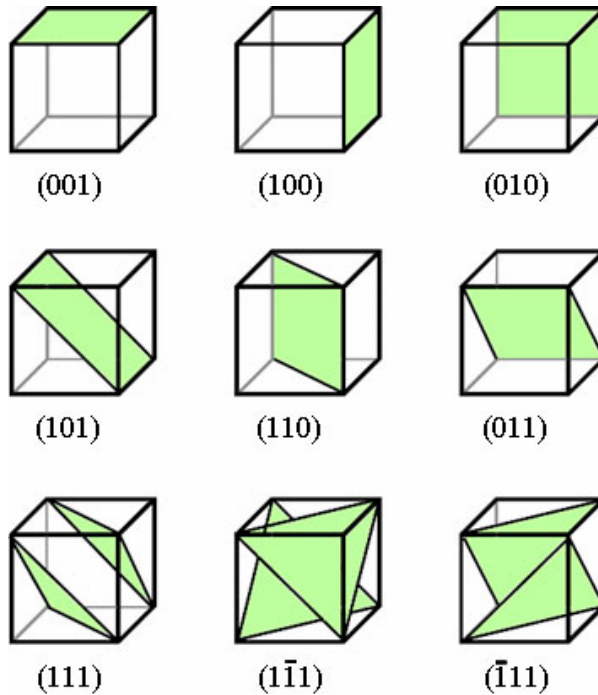
Solution:

The distance can be calculated from (??):

$$d_{211} = \frac{1}{\sqrt{2^2/2.3^2 + 1/3.4^2 + 1/4.6^2}} = 1.060 \text{ nm} \quad (1.1.29)$$

Problem 10.

Draw all 9 lattice planes and determine the Miller indexes in a simple cubic Bravais lattice.

Solution:

(Where is the origin of the 3 lattice vectors in the cubes?)

Problem 11.

Determine the possible diffraction angles for an X-ray of 10 keV from the (111) planes of a simple cubic lattice, if the lattice constant is $a = 5.3 \text{ \AA}$.

Solution:

Calculate the wavelength of the X-ray:

$$\mathcal{E} = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\mathcal{E}} = 1.24 \cdot 10^{-10} \text{ m}$$

To apply Bragg's law we need to calculate the distance of the lattice planes of Miller indices (111), which are planes going through 3 non adjacent corner of the cube. The distance of two such planes that intersects a primitive cell (see the figure in the previous example) is 3rd of the body diagonal:

$$d = \frac{1}{3}\sqrt{3} a = 0.305nm$$

The possible diffraction angles are determined by:

$$2 d \sin\theta = n \lambda$$

$$\sin\theta = n \frac{\lambda}{2d} = 0.20325 n$$

because $|\sin x| \leq 1$ $n = 1, 2, 3, 4$, i.e. the angles are

$$11.73^\circ, 23.99^\circ, 37.57^\circ, \text{ and } 54.39^\circ$$

Problem 12.

Aluminum has three valence electrons per atom, an atomic weight of 0.02698 kg/mol, a density of $2700kg/m^3$, and a conductivity of $3.54 \cdot 10^7$ S/m. Calculate the electron mobility in aluminum. Assume that all three valence electrons of each atoms are free.

Solution:

The number of aluminum atoms per m^3 is

$$\begin{aligned} n_a &= 6.0210^{23} \text{ atoms/mol} \cdot 1/0.02698 \text{ mol/kg} \cdot 2700 \text{ kg/m}^3 \\ &= 6.024 \cdot 10^{28} \text{ atoms/m}^3 \end{aligned}$$

Thus the electron density in aluminum is

$$n = 3 \cdot 6.024 \cdot 10^{28} \text{ atoms/m}^3 = 1.807 \cdot 10^{29} \text{ electron/m}^3$$

From (1.1.5)

$$\mu = \frac{\sigma}{ne} = \frac{3.54 \cdot 10^7}{1.807 \cdot 10^{29} \cdot 1.6022 \cdot 10^{-19}} = 1.22 \cdot 10^{-3} m/s$$

Problem 13.

What is the value of τ in silver at $t = 0^\circ C$ if the measured resistivity is $1.51 \cdot 10^{-8} \Omega m$?

Solution:

From (1.1.6), (1.1.7) using the definition of ρ

$$\tau = \frac{m_e}{\rho(T) n e^2} = \frac{m_e A}{L_A Z \rho_m e^2 \rho}$$

Silver has a single $5s^1$ electron so $Z = 1$ and the mass density is $\rho_m = 10.49 \text{ g/cm}^3 = 1.049 \cdot 10^4 \text{ kg/m}^3$, $A = 107.8682 \text{ g/mol} = 0.1078682 \text{ kg/mol}$ and $\rho(273K) = 1.51 \cdot 10^{-8} \Omega m$. After substitution

$$\underline{\underline{\tau = 4.013 \cdot 10^{-14} \text{ s}}}$$

Problem 14.

What is the quasi-free electron density in copper? Calculate the Fermi velocity and momentum too

Solution:

From Table 1.2 and formulas (1.1.12) and (1.1.10)

$$n_{copper} = \frac{(2m_e \mathcal{E}_F)^{3/2}}{3 \pi^2 \hbar^3} = 3.77 \cdot 10^{28} \frac{\text{electron}}{m^3}$$

$$v_F = 1.2 \cdot 10^6 \text{ m/s} = 0.004 c \quad k_F = 1.0 \cdot 10^{11} 1/m$$

Problem 15.

Let the dispersion relations of electrons in the valence and conduction bands near the band edges are approximated by the following functions:

$$\mathcal{E}_v(k) = -3.024 \cdot 10^{-20} \text{ eV m}^2 \cdot (k - 2.45 \cdot 10^8 \frac{1}{m})^2 + 13 \text{ eV}$$

$$\mathcal{E}_c(k) = 4.65 \cdot 10^{-20} \text{ J m}^2 \cdot k^2 + 11.9 \text{ eV}$$

Express the effective mass of electrons in units of the free electron mass $m_e = 9.1 \cdot 10^{-31} \text{ kg}$ in the valence and conduction bands!

Solution:

In 1D

$$\frac{1}{m_{eff}} = \frac{1}{\hbar^2} \frac{d^2 \mathcal{E}(k)}{dk^2}$$

The energies converted to Joule from eV are:

$$\mathcal{E}_v(k) = -4.845 \cdot 10^{-39} \text{ J m}^2 \cdot (k - 2.45 \cdot 10^8)^2 + 2.08 \cdot 10^{-18} \text{ J}$$

$$\mathcal{E}_c(k) = +7.450 \cdot 10^{-39} \text{ J m}^2 \cdot k^2 + 1.91 \cdot 10^{-18} \text{ J}$$

Both energy functions have the form $\mathcal{E} = a \cdot (k - b)^2 + d$. The second derivative of both functions gives twice the coefficient of the 2nd order terms ($a = \frac{d^2 \mathcal{E}}{dk^2}$), and so the electron effective masses in the conduction and valence bands are:

$$m_{eff}^{(c)} = -\frac{\hbar^2}{1.490 \cdot 10^{-38}} = -7.46 \cdot 10^{-31} [\text{kg}] = -0.819 m_e$$

$$m_{eff}^{(v)} = \frac{\hbar^2}{9.690 \cdot 10^{-39}} = 1.14 \cdot 10^{-30} [\text{kg}] = 1.260 m_e$$

Problem 16.

A rectangular copper strip 1.5 cm wide and 0.10 cm thick carries a current of 5.0 A. Find the Hall voltage for a 1.2 T magnetic field applied in a direction perpendicular to the strip. The molar mass and density of copper is $M = 0.0635 \text{ kg/mol}$, and $\rho = 8920 \text{ kg/m}^3$ respectively.

Solution:

The electron configuration of copper is $[Ar]3d^{10}4s^1$, therefore each copper atom provides one conduction electron. The number of electrons (or atoms) per unit volume (n) can be determined from the atomic mass and density of copper using the Avogadro number L_A :

$$n = \frac{L_A}{V_{mol}} = \frac{L_A \rho}{V_{mol} \rho} = \frac{L_A \rho}{M}$$

Substituting this into² (??) gives

$$\begin{aligned} U_H &= \frac{M I B}{L_A \rho q d} \\ &= \frac{0.0635 \text{ kg/mol} \cdot 5.0 \text{ A} \cdot 1.2 \text{ T}}{6.02 \cdot 10^{23} \text{ 1/mol} \cdot 8920 \text{ kg/m}^3 \cdot 1.60 \cdot 10^{-19} \text{ C} \cdot 10^{-3} \text{ m}} \\ U_H &= \underline{\underline{0.44 \mu V}} \end{aligned}$$

This is a very small voltage and it is hard to measure.

Problem 17.

Put a silicon (a semiconductor) sample of the same dimensions as in the previous example in magnetic field of the same magnitude. Determine the Hall voltage if it carries a current of 0.1 A! $n_{silicon} = 1.0 \cdot 10^{20} \text{ 1/m}^3$.

Solution:

Substituting I , B and n into equation (??) gives

$$U_H = \frac{I B}{e d n_{silicon}} = \underline{\underline{7.5 \text{ mV}}}$$

which is readily measured.

Problem 18.

Compare the electron excitation probabilities in silicon at room temperature and at $T = 450 \text{ K}$.

Solution:

Substituting into the probability formula: at room temperature

$$\mathcal{P}(300\text{K}) = 3.32 \cdot 10^{-19},$$

$${}^2 n_{copper} = \frac{L_A \rho}{M} = \frac{6.02 \cdot 10^{23} \text{ 1/mol} \cdot 8920 \text{ kg/m}^3}{0.0635 \text{ kg/mol}} = 8.46 \cdot 10^{28} \text{ 1/m}^3$$

while at $T = 450\text{K}$:

$$\mathcal{P}(450\text{K}) = 4.79 \cdot 10^{-13}$$

So the ratio of these probabilities is

$$\frac{\mathcal{P}(450\text{K})}{\mathcal{P}(300\text{K})} = 1.44 \cdot 10^6$$

i.e. the number of electrons available for conduction is about 10^6 larger at 450K than at 300 K. In the same range the increase in the resistivity because of lattice vibrations is about linear. As a consequence resistivity of Si decreases with increasing temperature, i.e. has a *negative thermal coefficient* contrary to metals. The same is true for all semiconductors.

Problem 19.

A rod of intrinsic Si is 1 cm long and has a diameter of 1mm. At room temperature, the intrinsic concentration in the silicon is $n_i = 1.5 \cdot 10^{16} \text{m}^{-3}$. The electron and hole mobilities are $\mu_e = 0.13 \text{m}^2 \text{V}^{-1} \text{s}^{-1}$ and $\mu_h = 0.05 \text{m}^2 \text{V}^{-1} \text{s}^{-1}$ respectively. Calculate the conductivity σ of silicon and the resistance R of the rod.

Solution:

$$\sigma = n_i e (\mu_e + \mu_h) = 4.33 \cdot 10^{-4} \text{1}/\Omega \text{m}$$

$$R = \frac{l}{\sigma d^2 \pi / 4} = 29.4 \text{M}\Omega$$

Problem 20.

Calculate the effective densities of states in the conduction and valence bands of germanium and silicon at 300 K.

Solution:

Substitute the effective masses for the density of states from Table 1.1 into (1.1.16) and (1.1.17).

cm^{-3}	Ge	Si
$N_c(300\text{K})$	$1.02 \cdot 10^{19}$	$2.81 \cdot 10^{19}$
$P_v(300\text{K})$	$5.64 \cdot 10^{18}$	$1.83 \cdot 10^{19}$

Problem 21.

A rod of n-type extrinsic Si is 1 cm long and has a diameter of 1mm. At room temperature, the donor concentration is $5 \cdot 10^{14} \text{atom}/\text{cm}^3$ and this corresponds to 1 impurity for 10^8 Si atoms. A steady $2\mu\text{A}$ current is flowing through the bar. Determine the electron and hole concentrations, the conductivity and the voltage across the rod. The intrinsic electron concentration in silicon is³ $n_i = 1.01 \cdot 10^{10} \text{m}^{-3}$. The electron mobility is $\mu_e = 0.13 \text{m}^2 \text{V}^{-1} \text{s}^{-1}$.

Solution:

³The previously accepted value before 1991 was $1.45 \cdot 10^{10} \text{m}^{-3}$

n-type: $N_a \approx 0$. From (1.1.13) $n = N_d = 5 \cdot 10^{20} \text{1/m}^3$,

$$p_v = \frac{n_i^2}{N_d} = 4.5 \cdot 10^{13} \text{1/m}^3$$

From (1.1.8) and using that $p_v \ll n_c$

$$\sigma = en_c \mu_e = 0.104 \text{ S/m}$$

The voltage across the rod:

$$U = \frac{I l}{\sigma A} = 0.12 \text{ V}$$

Problem 22.

Determine the ratio of conduction electrons from P dopants in Si to the intrinsic electron concentration at the following temperatures: room temperature, 100°C and 500°C ! Assume that the concentration of P is about 10^{-8} times smaller than that of the Si atoms. Is it possible for the intrinsic electron concentration to become larger than the one due to the dopants and if so at which temperature?

Solution:

a)

P is a donor atom, therefore the ratio of the conduction electrons from P and from the valence band can be calculated using the Fermi energy for donor states. and neglecting the factor $\ln\left(\frac{m_c}{m_h}\right)$:

$$\begin{aligned} \mathcal{E}_F^{(d)} &= (\mathcal{E}_c - \mathcal{E}_d)/2, & \mathcal{E}_F &= (\mathcal{E}_c - \mathcal{E}_v)/2 = \mathcal{E}_g/2 \\ \frac{n_c^{(d)}(T)}{N_d} &= e^{-\frac{\mathcal{E}_F^{(d)}}{k_B T}}, & \frac{n_c(T)}{n} &= e^{-\frac{\mathcal{E}_F}{k_B T}} \\ \frac{n_c^{(d)}(T)}{n_i(T)} &= \frac{N_d}{n} \cdot \frac{e^{-\frac{\mathcal{E}_F^{(d)}}{k_B T}}}{e^{-\frac{\mathcal{E}_F}{k_B T}}} = \frac{N_d}{n} \cdot e^{-\frac{\mathcal{E}_F^{(d)} - \mathcal{E}_F}{k_B T}} = \frac{N_d}{n} \cdot e^{-\frac{(\mathcal{E}_c - \mathcal{E}_d) - \mathcal{E}_g}{2k_B T}} \end{aligned}$$

Here $\frac{N_d}{n}$ is about 10^{-8} . $k_B T$ at room temperature (300 K) equals to 0.0258 eV. At other T temperatures $k_B T = k_B T^{(room)} \cdot \frac{T}{T^{(room)}} = 0.0258 \text{ eV} \cdot \frac{T}{300 \text{ K}}$, therefore at 100°C (373K) and 500°C (773K) $k_B T$ is $1.243 \cdot 0.0258 \text{ eV} = 3.208 \cdot 10^{-2} \text{ eV}$ and $2.577 \cdot 0.0258 \text{ eV} = 6.648 \cdot 10^{-2} \text{ eV}$ respectively. Using these numbers

$$\begin{aligned} \frac{n_c^{(d)}(T)}{n_i(T)} &= 10^{-8} \cdot e^{-\frac{(0.046 - 1.12) \text{ eV}}{2 \cdot 0.0258 \text{ eV}}} = 10^{-8} \cdot e^{\frac{1.074}{2 \cdot 0.0258}} = \underline{\underline{4.375 \cdot 10^{10}}} \\ \frac{n_c^{(d)}(373\text{K})}{n_i(373\text{K})} &= 10^{-8} \cdot e^{\frac{1.074}{2 \cdot 0.0321}} = \underline{\underline{9.605 \cdot 10^6}} \\ \frac{n_c^{(d)}(773\text{K})}{n_i(773\text{K})} &= 10^{-8} \cdot e^{\frac{1.074}{2 \cdot 0.0665}} = \underline{\underline{1.707 \cdot 10^{-1}}} \end{aligned}$$

b)

The intrinsic electron concentration reaches $n_c^{(d)}$ when

$$10^{-8} \cdot e^{\frac{1.074 \text{ eV}}{2 \cdot k_B \cdot T}} = 1 \quad \Rightarrow \quad \frac{1.074 \text{ eV}}{2 \cdot k_B \cdot T} = \ln(10^8)$$

From which

$$k_B T = \frac{1.074 \text{ eV}}{36.84} = 0.0583 \text{ eV} \quad \Rightarrow \quad T = \frac{0.0583 \text{ eV}}{0.0258 \text{ eV}} \cdot T^{(room)} = \underline{\underline{678 \text{ K}}}$$

Problem 23.

Calculate the total voltage difference (the *built in potential*) between the n-type and p-type part for a uniformly doped Silicon p-n junction with $N_d = N_a = 10^{17} \text{ cm}^{-3}$ at room temperature. The intrinsic carrier density is $1.45 \cdot 10^{14} \text{ m}^{-3}$ Will the built-in potential increase or decrease with an increase in temperature?

Solution:

From formula (1.1.24)

$$V_{p-n} \equiv \Delta\varphi = \frac{1}{e} k_B T \ln \left(\frac{N_d N_a}{n_i^2} \right) \quad (1.1.30)$$

$$\begin{aligned} V_{p-n} &= 1.38 \cdot 10^{-23} \left[\frac{\text{J}}{\text{K}} \right] 300 [\text{K}] \frac{1}{1.6022 \cdot 10^{-19} [\text{As}]} \ln \left(\frac{10^{17} 10^{17}}{(1.45 \cdot 10^{14})^2} \right) \\ &= 0.34 \text{ V} \end{aligned}$$

Substituting back the formula of $n_i^2(T)$ from (1.1.23):

$$n_i^2 = N_c(T) P_v(T) e^{-\frac{\mathcal{E}_g}{k_B T}}$$

and $N_c(T)$ = and $P_v(T)$, from (1.1.3)

$$N_c(T) = 2.5 \cdot 10^{19} \left(\frac{m_c}{m_e} \right)^{\frac{3}{2}} \left(\frac{T}{300 \text{ K}} \right)^{\frac{3}{2}}$$

$$P_v(T) = 2.5 \cdot 10^{19} \left(\frac{m_h}{m_e} \right)^{\frac{3}{2}} \left(\frac{T}{300 \text{ K}} \right)^{\frac{3}{2}}$$

we find that V_{p-n} is of the form:

$$e V_{p-n} = A \cdot T - B \cdot T \cdot \ln T + \mathcal{E}_g$$

When T increases the change in the term containing $-T \cdot \ln T$ is larger than the change in the term containing $T \rightarrow$ the voltage decreases.

Problem 24.

Determine the widths d_n and d_p and the electrical field strength for the Si p-n junction of the previous example. The relative permittivity of Si is $\epsilon_r = 16.0$.

Solution:

We may write (1.1.19) in a more convenient form:

$$\begin{aligned} d_n &= 5257 \sqrt{\frac{N_a}{N_d(N_d + N_a)} \frac{\epsilon_r \Delta\varphi}{2}} \text{ [nm]} \\ d_p &= 5257 \sqrt{\frac{N_d}{N_a(N_d + N_a)} \frac{\epsilon_r \Delta\varphi}{2}} \text{ [nm]} \end{aligned} \tag{1.1.31}$$

Substituting the data from the previous example gives $d_n = d_p = 425.8$ nm. The magnitude of E is $\Delta\varphi/(d_n + d_p) = 0.82\text{V}/4.258 \cdot 10^{-7} \text{ m} = 1.93 \cdot 10^7 \text{ V/m}$.

Problem 25.

The susceptibility of a ferromagnetic material is $\chi = 0.0116$ at $T = 1100$ K and $\chi = 0.0042$ at $T = 1200$ K. What material is it? What is the value of C for this material?

Solution:

Calculate its Curie temperature first. Using (1.1.26) for both susceptibility we get two equations for the two unknown, from which both T_c and C (and therefore λ too) can be determined

$$\begin{aligned} \chi_1 &= \frac{C}{T_1 - T_c} &\Rightarrow & C = \chi_1 \cdot (T_1 - T_c) \\ \chi_2 &= \frac{C}{T_2 - T_c} &\Rightarrow & C = \chi_2 \cdot (T_2 - T_c) \\ \chi_1 \cdot (T_1 - T_c) &= \chi_2 \cdot (T_2 - T_c) \\ T_c &= \frac{\chi_1 \cdot T_1 - \chi_2 \cdot T_2}{\chi_1 - \chi_2} = 1043 \text{ K} \end{aligned}$$

Using Table 1.4 we find the material is iron. Furthermore from the equations above we get the value of $C = 0.66$.